5.2

TABLE Eigenfunctions $R_\nu(\lambda_n, r)$, the Norm $N(\lambda_n)$, and the Eigenvalues λ_n of the Differential Equation

$$\frac{d^2R_\nu}{dr^2} + \frac{1}{r}\frac{dR_\nu}{dr} + \left(\lambda^2 - \frac{\nu^2}{r^2}\right)R_\nu = 0 \text{ in } 0 \le r < b$$

Subject to the Boundary Conditions Shown

Case No.	Boundary Condition at $r = b$	$R_\nu(\lambda_n, r)$	$\frac{1}{N(\lambda_n)}$	Eigenvalues λ_n Are the Positive Roots of
1	$\frac{dR_\nu}{dr} + HR_\nu = 0$	$J_\nu(\lambda_n r)$	$\frac{2}{J_\nu^2(\lambda_n b)} \cdot \frac{\lambda_n^2}{b^2(H^2+\lambda_n^2) - \nu^2}$	$\left.\frac{dJ_\nu(\lambda_n r)}{dr}\right\|_{r=b} + HJ_\nu(\lambda_n b) = 0$
2	$\frac{dR_\nu}{dr} = 0$	$J_\nu(\lambda_n r)^a$	$\frac{2}{J_\nu^2(\lambda_n b)} \cdot \frac{\lambda_n^2}{b^2\lambda_n^2 - \nu^2}$ [a]	$\left.\frac{dJ_\nu(\lambda_n r)}{dr}\right\|_{r=b} = 0^a$
3	$R_\nu = 0$	$J_\nu(\lambda_n r)$	$\frac{2}{b^2 J_{\nu+1}^2(\lambda_n b)}$	$J_\nu(\lambda_n b) = 0$

[a] For this particular case $\lambda_0 = 0$ is also an eigenvalue with $\nu = 0$; then the corresponding eigenfunction is $R_0 = 1$ and the norm $1/N(\lambda_0) = 2/b^2$. $H = h/k$.