5.3

TABLE **Solution $R_\nu(\lambda_n, r)$, the Norm $N(\lambda_n)$, and the Eigenvalues λ_n of the Differential Equation**

$$\frac{d^2 R_\nu(r)}{dr^2} + \frac{1}{r}\frac{dR_\nu(r)}{dr} + \left(\lambda^2 - \frac{\nu^2}{r^2}\right) R_\nu(r) = 0 \text{ in } a < r < b$$

Case No.	Boundary Condition at $r = a$	Boundary Condition at $r = b$	$R_\nu(\lambda_n, r)$[a] and $\frac{1}{N(\lambda_n)}$	λ_n Values Are the Positive Roots of[a]
1	$\frac{dR_\nu}{dr} = 0$	$\frac{dR_\nu}{dr} = 0$	$R_\nu(\lambda_n, r) = J_\nu(\lambda_n r)Y'_\nu(\lambda_n a) - J'_\nu(\lambda_n a)Y_\nu(\lambda_n r)$ $\frac{1}{N(\lambda_n)} = \frac{\pi^2}{2} \frac{\lambda_n^2 J_\nu'^2(\lambda_n b)}{\left[1 - \left(\frac{\nu}{\lambda_n b}\right)^2\right] J_\nu'^2(\lambda_n a) - \left[1 - \left(\frac{\nu}{\lambda_n a}\right)^2\right] J_\nu'^2(\lambda_n b)}$	$J'_\nu(\lambda_n a)Y'_\nu(\lambda_n b)$ $- J'_\nu(\lambda_n b)Y'_\nu(\lambda_n a) = 0$[b]
2	$\frac{dR_\nu}{dr} = 0$	$R_\nu = 0$	$R_\nu(\lambda_n, r) = J_\nu(\lambda_n r)Y'_\nu(\lambda_n a) - J'_\nu(\lambda_n a)Y_\nu(\lambda_n r)$ $\frac{1}{N(\lambda_n)} = \frac{\pi^2}{2} \frac{\lambda_n^2 J_\nu^2(\lambda_n b)}{J_\nu'^2(\lambda_n a) - \left[1 - \left(\frac{\nu}{\lambda_n a}\right)^2\right] J_\nu^2(\lambda_n b)}$	$J_\nu(\lambda_n b)Y'_\nu(\lambda_n a)$ $- J'_\nu(\lambda_n a)Y_\nu(\lambda_n b) = 0$
3	$R_\nu = 0$	$\frac{dR_\nu}{dr} = 0$	$R_\nu(\lambda_n, r) = J_\nu(\lambda_n r)Y_\nu(\lambda_n a) - J_\nu(\lambda_n a)Y_\nu(\lambda_n r)$ $\frac{1}{N(\lambda_n)} = \frac{\pi^2}{2} \frac{\lambda_n^2 J_\nu'^2(\lambda_n b)}{\left[1 - \left(\frac{\nu}{\lambda_n b}\right)^2\right] J_\nu^2(\lambda_n a) - J_\nu'^2(\lambda_n b)}$	$J'_\nu(\lambda_n b)Y_\nu(\lambda_n a)$ $- J_\nu(\lambda_n a)Y'_\nu(\lambda_n b) = 0$
4	$R_\nu = 0$	$R_\nu = 0$	$R_\nu(\lambda_n, r) = J_\nu(\lambda_n r)Y_\nu(\lambda_n a) - J_\nu(\lambda_n a)Y_\nu(\lambda_n r)$ $\frac{1}{N(\lambda_n)} = \frac{\pi^2}{2} \frac{\lambda_n^2 J_\nu^2(\lambda_n b)}{J_\nu^2(\lambda_n a) - J_\nu^2(\lambda_n b)}$	$J_\nu(\lambda_n a)Y_\nu(\lambda_n b)$ $- J_\nu(\lambda_n b)Y_\nu(\lambda_n a) = 0$

[a] We note here that the prime notation denotes differentiation with respect to the entire argument. For example, $J'_\nu(\lambda b) = dJ_\nu(\lambda b)/d(\lambda b)$.

[b] For this particular case $\lambda_0 = 0$ is also an eigenvalue with $\nu = 0$; the corresponding eigenfunction is $R_0(\lambda_0, r) = 1$ and the norm $1/N(\lambda_0) = 2/(b^2 - a^2)$.

Source: From reference 12.